

Analogue of the quantum total probability rule from Paraconsistent bayesian probability theory

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We derive an analogue of the quantum total probability rule by constructing a probability theory based on paraconsistent logic. Bayesian probability theory is constructed upon classical logic and a desiderata, that is, a set of desired properties that the theory must obey. We construct a new probability theory following the desiderata of Bayesian probability theory but replacing the classical logic by paraconsistent logic. This class of logic has been conceived to handle eventual inconsistencies or contradictions among logical propositions without leading to the trivialisation of the theory. Within this Paraconsistent bayesian probability theory it is possible to deduce a new total probability rule which depends on the probabilities assigned to the inconsistencies. Certain assignments of values for these probabilities lead to expressions identical to those of Quantum mechanics, in particular to the quantum total probability rule obtained via symmetric informationally complete positive-operator valued measure.

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According to the bayesian approach, probability theory is an extension of logic [1]. Probabilities are a measure, assigned by an agent to the plausibility of a proposition conditional on the truth of a priori information [2]. In the limit of complete knowledge, when probabilities achieve extremal values, the rules for handling probabilities become those of deductive classical logic.

Bayesian probability theory (BPT) has been successfully applied in a wide range of research areas such as for instance genetics [3] and cosmology [4]. In spite of this, there are phenomena which are beyond the scope of BPT. A noteworthy example is Quantum mechanics, where probabilities do not follow, in general, the total probability rule of BPT. Quantum states can be expressed as convex combinations of operators, in a symmetrically informationally complete-positive operator valued measure (SIC-POVM), weighted by probabilities. This representation, together with Born's rule, allows us to calculate the probability of the outcomes associated to any other positive operator-valued measure (POVM), which leads to the quantum total probability rule. This turns out to be related to the total probability rule of BPT through rescaling and translation operations.

Arises thus naturally the question whether we can modify BPT in such a way that it provides us a new total probability rule which encompasses as particular cases the total probability rule of BPT and its quantum counterpart. Here, we attempt to answer this question by constructing a probability theory based on paraconsistent logic. Paraconsistent logics [5–7] are designed to handle theories in which inconsistencies or contradictions might arise without leading to a trivialisation or logical explosion. Thereby, this new Paraconsistent bayesian probability theory (PBPT) requires the assignation of

probabilities to the occurrence of contradictions. These enter in the new total probability rule and, depending on their values, it is possible to recover the total probability rule of BPT and its quantum version. Let us note that we do not seek a new interpretation of Quantum mechanics, but a new probability theory upon which we can build Quantum mechanics.

In BPT the basic product and sum rules for combining probabilities are given by

$$P(A, B|I) = P(A|I)P(B|A, I) = P(B|I)P(A|B, I) \quad (1)$$

and

$$P(A|I) + P(\bar{A}|I) = 1, \quad (2)$$

respectively. Symbols A , B and I represent propositions asserting that something is true and a bar over a proposition indicates its logical negation. A proposition to the right of a vertical bar is assumed to be true and a comma separating two propositions indicates the logical conjunction. Sum of two propositions indicates the logical disjunction. Thus $P(A|B, I)$ means the probability that A is true conditional on the truth of both B and I . Bayes rule follows by rearranging the two terms at the right hand side of the product rule, that is

$$P(B|A, I) = \frac{P(B|I)P(A|B, I)}{P(A|I)}. \quad (3)$$

The total probability rule, which follows from a marginalisation process over a set of complete, mutually exclusive propositions $\{A_i\}$, is given by

$$P(B|I) = \sum_i P(A_i|I)P(B|A_i, I). \quad (4)$$

Sum and product rules can be deduced from classical logic by means of a desiderata [1], that is, a set of desirable properties that a theory for plausible reasoning or inference should satisfy. In this regard, the desiderata does not assert anything to be true. The desiderata is: (i) degrees of plausibility are represented by real numbers, (ii) as new information supporting the truth of a proposition is supplied, the number which represents the plausibility will increase continuously and monotonically and achieve the deductive limit where appropriate, (iiia) if a conclusion can be reasoned out in more than one way, every possible way must lead to the same result (structural consistency), (iiib) the theory must take account of all information, provided it is relevant to the question (propriety), and (iiic) equivalent states of knowledge must be represented by equivalent plausibility assignments (Jaynes consistency).

In order to construct the PBPT we choose to maintain the desiderata but change the underlying logic. Instead of basing our construction on classical logic we resort to a paraconsistent logic, in particular the \mathcal{C}_1 propositional calculus [8]. Paraconsistent logics are logics in which theories can be inconsistent but non-trivial. A trivial theory is one, such that everything expressed in its language can be proved. In classical logic, any inconsistency entails triviality, that is: $(A, \bar{A}) \vdash B$ (where \vdash indicates syntactic consequence or formal deduction) for any formulas A and B , which is not true in paraconsistent logics.

A theory T , whose underlying logic is L and whose language is \mathcal{L} , is *inconsistent* if there is a formula α (an admissible expression of its language) such that both α and $\bar{\alpha}$ are theorems (formulas deduced from the axioms of the theory by means of its rules of inference) of T , otherwise T is *consistent*. An expression of the form $\alpha, \bar{\alpha}$ is called *contradiction*. T is *trivial* if all formulas of \mathcal{L} are theorems of T ; otherwise, T is *non-trivial*. The logic L is *paraconsistent* if it can be the underlying logic of inconsistent but non-trivial theories. The propositional calculus \mathcal{C}_1 contains the usual connectives: implication ($\alpha \rightarrow \beta$), conjunction (α, β), disjunction ($\alpha + \beta$) and negation ($\bar{\alpha}$).

Two important properties of \mathcal{C}_1 are: (i) in general, the principle of non-contradiction does not hold, and (ii) from two contradictory propositions, that is one being the negation of the other, it is not possible to deduce any arbitrary third proposition. This latter property ensures that the presence of contradictions by no means entail the trivialisation of the theory. Within \mathcal{C}_1 we define the *non-contradictoriness* of a proposition α by $\alpha^\circ = \bar{\alpha}, \bar{\alpha}$. If α° is true then for α holds the principle of non-contradiction and we said that α is *non-contradictory*. Otherwise we say that α is *contradictory*. The proposition $\bar{\alpha}^\circ$ is called the *contradictoriness* of α . Other important definition is the *strong negation* of α by $\bar{\alpha}^* = \bar{\alpha}, \alpha^\circ$. Two important results follow from these definitions: (a) for any α we have that $\vdash (\alpha^\circ)^\circ$ (Arruda's theorem [8]) and (b) for any

α and β the connectives $\alpha \rightarrow \beta$, $\alpha + \beta$, $\bar{\alpha}^*$ and α, β satisfy all schemas and inference rules of classical propositional calculus.

Let us now construct the PBPT. We will keep the desiderata used to construct BPT. However, for the desiderata (ii) the deductive limit is that of the valid schemas of \mathcal{C}_1 -logic, which is the same as the classical logic when the statements are non-contradictory. We must also consider that the contradictoriness of a statement is a new relevant information for the plausibility of the statement. In BPT the product rule is deduced [9] by analysing all possible functional forms for the probability. All but one of these are demonstrated to be inadequate by studying several particular cases such as $A = B$, $I = A$, $I = \bar{A}$, etc. These particular cases are included in \mathcal{C}_1 -logic, provided that the contradictoriness of the statement is a relevant information. Thus, by desiderata (iiib) a proposition like $I = \bar{A}$ should read as $I = \bar{A}, I$ or $I = \bar{A}, \bar{A}^\circ$ or $I = \bar{A}, A^\circ$ and the last one rules out the same functional form as $I = \bar{A}$. Since the deduction of the product rule Eq. (1) does not involve any particular schema (only properties of the connectives, like the symmetry of conjunction) it also holds for \mathcal{C}_1 -logic.

For any non-contradictory formula in \mathcal{C}_1 all schemas from classical logic are also valid. Thus, the results of BPT are also valid for these. Consequently, we have

$$P(A|A^\circ, I) + P(\bar{A}|A^\circ, I) = 1. \quad (5)$$

Following (a) we have that any formula's non-contradictoriness is always non-contradictory. Thus

$$P(A^\circ|I) + P(\bar{A}^\circ|I) = 1, \quad (6)$$

and according to (b) we have for the strong negation that

$$P(A|I') + P(\bar{A}^*|I') = 1. \quad (7)$$

We can now deduce the sum rule of PBPT. Let us start with Eq. (5) and multiply it by $P(A^\circ|I)$, that is,

$$P(A^\circ|I)P(A|A^\circ, I) + P(A^\circ|I)P(\bar{A}|A^\circ, I) = P(A^\circ|I). \quad (8)$$

Making use of the product rule Eq. (1) we obtain

$$P(A|I)P(A^\circ|A, I) + P(\bar{A}|I)P(A^\circ|\bar{A}, I) = P(A^\circ|I). \quad (9)$$

Using Eq. (6) and the product rule the previous expression becomes

$$P(A|I) + P(\bar{A}|I) - P(\bar{A}^\circ, A|I) - P(\bar{A}^\circ, \bar{A}|I) = 1 - P(\bar{A}^\circ|I). \quad (10)$$

Considering that by the definition of contradictoriness we have $\bar{A}^\circ, A = \bar{A}^\circ$ and $\bar{A}^\circ, \bar{A} = \bar{A}^\circ$ we finally obtain the sum rule of PBPT

$$P(A|I) + P(\bar{A}|I) - P(\bar{A}^\circ|I) = 1. \quad (11)$$

Clearly, the PBPT sum rule, not only involves a proposition and its negation, but also its contradictoriness. In

general $P(\bar{A}^\circ)$ does not vanish and thus $P(A) \neq 1 - P(\bar{A})$. Furthermore, the probability of a contradiction works as a negative probability when thinking about it as an statement itself. Consequently, the addition of $P(A|I)$ and $P(\bar{A}|I)$ might be larger than one. In particular, when we known with certainty that A is contradictory, that is, $P(\bar{A}^\circ) = 1$, then the sum rule demands the assignment $P(A) = 1 = P(\bar{A})$.

From Eq. (7) and using Eq. (6) together with product and sum rules Eqs. (1) and (11) we can deduce the extended sum rule which involves the probability of the disjunction of propositions A and B , that is,

$$P(A + B|I) = P(A|I) + P(B|I) - P(A, B|I). \quad (12)$$

This is the same rule as in the case of BPT.

When analysing the probability of an statement, such as A , we can also reason based on the knowledge about their non-contradictory or contradictory parts, that is, on $\tilde{A} = A, A^\circ$ or A, \bar{A}° , respectively. The propriety desiderata (iiib) demands that this information must play a relevant role in the assignment of probability. Since $A, \bar{A}^\circ = \bar{A}^\circ$, a reasoning based on the contradictory part never reaches the deductive classical limit (i.e. $I = A^\circ, I$), thus desiderata (iiib) suggests us to consider the non-contradictoriness of our data. For instance, we can deduce the probability $P(\tilde{A} + \tilde{B}|I)$ for the conjunction of the mutually exclusive non-contradictory parts of A and B , that is when the proposition A, A°, B, B° does not hold. Under this condition the extended sum rule Eq. (12) leads to

$$P(\tilde{A} + \tilde{B}|I) = P(\tilde{A}|I) + P(\tilde{B}|I), \quad (13)$$

which with the product rule becomes

$$P(\tilde{A} + \tilde{B}|I) = P(A|I)P(A^\circ|A, I) + P(B|I)P(B^\circ|B, I). \quad (14)$$

Eq. (6) allows us to transform the terms involving the non-contradictoriness in the previous expression into terms involving the contradictoriness, that is

$$P(\tilde{A} + \tilde{B}|I) = P(A|I)[1 - P(\bar{A}^\circ|A, I)] + P(B|I)[1 - P(\bar{B}^\circ|B, I)]. \quad (15)$$

Employing once again the product rule and $A, \bar{A}^\circ = \bar{A}^\circ$ and $B, \bar{B}^\circ = \bar{B}^\circ$ we finally obtain the expression

$$P(\tilde{A} + \tilde{B}|I) = P(A|I) + P(B|I) - [P(\bar{A}^\circ|I) + P(\bar{B}^\circ|I)]. \quad (16)$$

Thus, the extended sum rule for two mutually exclusive non-contradictory parts turns out to be fundamentally different to Eq. (12). The last term in Eq. (16) corresponds to the negative of the total probability that the statements are contradictory. Equivalently, when considering the sum rule of PBPT Eq. (16) becomes

$$P(\tilde{A} + \tilde{B}|I) = 2 - P(\bar{A}|I) - P(\bar{B}|I). \quad (17)$$

Eq. (16) can be easily extended to the conjunction of N propositions A_k (with $k = 1, \dots, N$) with mutually exclusive non-contradictory parts, that is,

$$P(\sum_k \tilde{A}_k|I) = \sum_k P(A_k|I) - \sum_k P(\bar{A}_k^\circ|I). \quad (18)$$

This later result allows us to calculate the probability of the conjunction between propositions B and $\sum_k \tilde{A}_k$. The product rules leads us to

$$P(B|\sum_k \tilde{A}_k, I)P(\sum_k \tilde{A}_k|I) = P(B, \sum_k \tilde{A}_k|I), \quad (19)$$

which when combined with Eq. (18) leads to

$$P(B|\sum_k \tilde{A}_k|I) = \frac{P(B, \sum_k \tilde{A}_k|I)}{\sum_k [P(A_k|I) - P(\bar{A}_k^\circ|I)]}. \quad (20)$$

Using Eq. (13) we obtain

$$P(B|\sum_k \tilde{A}_k, I) = \frac{\sum_k P(\tilde{A}_k|I)P(B|\tilde{A}_k, I)}{\sum_k [P(A_k|I) - P(\bar{A}_k^\circ|I)]}. \quad (21)$$

Decomposing a proposition A_k into its contradictory and non-contradictory parts we have $A_k = \tilde{A}_k + A_k, \bar{A}_k^\circ$, or equivalently $A_k = \tilde{A}_k + \bar{A}_k^\circ$. Using the extended sum rule Eq. (12) we obtain

$$P(A_k|I) = P(\tilde{A}_k|I) + P(\bar{A}_k^\circ|I) - P(\tilde{A}_k, \bar{A}_k^\circ|I). \quad (22)$$

The last term vanishes since proposition A_k° is non-contradictory. Thereby, we have

$$P(A_k|I) = P(\tilde{A}_k|I) + P(\bar{A}_k^\circ|I). \quad (23)$$

This result allows us to cast Eq. (21) as

$$P(B|\sum_k \tilde{A}_k, I) = \frac{\sum_k P(A_k|I)P(B|\tilde{A}_k, I)}{\sum_k [P(A_k|I) - P(\bar{A}_k^\circ|I)]} - \frac{\sum_k P(\bar{A}_k^\circ|I)P(B|\tilde{A}_k, I)}{\sum_k [P(A_k|I) - P(\bar{A}_k^\circ|I)]}, \quad (24)$$

which is the total probability rule of PBPT. Let us now consider the propositions A_k to be non-contradictory, that is, our a priori information is $I = A_1^\circ, A_2^\circ, \dots, A_N^\circ, I$. Since propositions $\{\tilde{A}_k\}$ are mutually exclusive, we also have that propositions $\{A_k\}$ are mutually exclusive. Assuming the completeness of the set $\{A_k\}$ we have that $P(\sum_k A_k|I) = 1 = \sum_k P(A_k|I)$. Thereby, the second term at the right hand side of Eq. (24) vanishes and the denominator becomes 1. We obtain

$$P(B|\sum_k A_k, I) = \sum_k P(A_k|I)P(B|A_k, I). \quad (25)$$

Thus, the total probability rule of BPT given by Eq. (4) is contained within PBPT. Total probability rule

of PBPT Eq. (24) also allows to calculate probabilities $P(\bar{B}|\sum_k \tilde{A}_k, I)$ and $P(\bar{B}^\sigma|\sum_k \tilde{A}_k, I)$, which together with probability $P(B|\sum_k \tilde{A}_k, I)$ also obey the sum rule Eq. (11).

In order to obtain the total probability rule of BPT we assumed the non-contradictoriness of propositions $\{A_k\}$. We can now show that the total probability rule Eq. (24) contains the quantum total probability rule for a set of propositions endowed with a particular structure of contradictoriness. Let us first briefly review the quantum total probability rule. Quantum states of a d -dimensional quantum system are described by unit-trace positive semidefinite linear operators ρ which act onto the Hilbert space \mathcal{H}_d . States can also be described by a collection of probabilities with the help of a SIC-POVM [10]. This is composed of d^2 subnormalised rank one projectors $\{\Pi_k/d\}$ with Hilbert-Schmidt products given by $Tr(\Pi_k \Pi_l) = (d\delta_{k,l} + 1)/(d+1)$. These operators generate the representation for quantum states

$$\rho = \frac{d+1}{d} \sum_{k=0}^{d^2-1} Q(\Pi_k|I) \Pi_k - \frac{1}{d} \sum_{k=0}^{d^2-1} \Pi_k, \quad (26)$$

where $Q(\Pi_k|I) = Tr(\Pi_k \rho)$ is Born's rule [11]. This set of probabilities contains all the information about the system required to predict the outcomes of possible experiments. Using this representation for quantum states we can calculate the transition probability associated to any other state Σ , that is

$$Q(\Sigma|\rho) = \frac{d+1}{d} \sum_{k=0}^{d^2-1} Q(\Pi_k|I) Q(\Sigma|\Pi_k) - \frac{1}{d} \sum_k Q(\Sigma|\Pi_k). \quad (27)$$

This expression is the quantum total probability rule and allows us to predict the transition probability to another state Σ from our knowledge of the initial state of the system, given by the sets of probabilities $\{Q(\Pi_k|I)\}$, and the probabilities $\{Q(\Sigma|\Pi_k)\}$. In this regard, the quantum total probability rule is equivalent to Born's rule [12].

Quantum and paraconsistent total probability rules contain the subtraction of two non-negative terms being one of them a constant. With the introduction of a particular set of propositions, which emulates certain properties of a SIC-POVM, we can deduce a paraconsistent total probability rule that exhibits a stronger similarity with the quantum total probability rule. From Eq. (23) we have

$$\sum_i P(A_i|I) = \sum_i P(\tilde{A}_i|I) + \sum_i P(\bar{A}_i^\sigma|I). \quad (28)$$

We also have that

$$P(\sum_i A_i|I) = 1 = P(\sum_i \tilde{A}_i + \sum_i \bar{A}_i^\sigma|I). \quad (29)$$

We now assume that propositions A_k are all simultaneously contradictory or non-contradictory. In this case we require a single proposition to signal this property, that is $\bar{A}_1^\sigma = \bar{A}_2^\sigma = \dots = \bar{A}_N^\sigma = \bar{A}^\sigma$. Thereby, we have that $P(\sum_i \tilde{A}_i + \bar{A}^\sigma|I) = 1$, which with the help of the extended sum rule Eq. (12) leads to

$$\sum_i P(\tilde{A}_i|I) + P(\bar{A}^\sigma|I) = 1. \quad (30)$$

Inserting Eq. (30) into Eq. (28) we obtain

$$P(\bar{A}^\sigma|I) = \frac{1}{N-1} \sum_i P(A_i|I) - \frac{1}{N-1}. \quad (31)$$

This latter equation allows us to write Eq. (24) in the form

$$P(B|\sum_k \tilde{A}_k, I) = \frac{N-1}{N-S(I)} \sum_k P(A_k|I) P(B|\tilde{A}_k, I) - \frac{S(I)-1}{N-S(I)} \sum_k P(B|\tilde{A}_k, I), \quad (32)$$

with $S(I) = \sum_k P(A_k|I)$. We can now link both total probability rules Eqs. (32) and (27). In our model of propositions we now make the number of propositions equal to the number of elements in the SIC-POVM, that is $N = d^2$, and assign the value $\sum_i P(A_i|I) = d$. Entering these values in Eq. (32) we obtain the expression

$$P(B|\sum_{k=0}^{d^2-1} \tilde{A}_k, I) = \frac{d+1}{d} \sum_{k=0}^{d^2-1} P(A_k|I) P(B|\tilde{A}_k, I) - \frac{1}{d} \sum_{k=0}^{d^2-1} P(B|\tilde{A}_k, I). \quad (33)$$

The similarity between total probability rules Eqs. (27) and (33) is striking. In Quantum mechanics the set $\{Q(\Pi_k|I)\}$ provides all the information needed to associate the system to a unique quantum state ρ through Eq. (26). In PBPT the set $\{P(A_k|I)\}$ represents our state of knowledge about the truth values of the set $\{A_k\}$ of complete, possibly contradictory, and mutually exclusive propositions. In Quantum mechanics the quantities $\{Q(\Sigma|\Pi_k)\}$ represent the probabilities of projecting onto state Σ provided that we know with certainty that the system is described by the state Π_k . This set characterises the state Σ with respect to the members of the SIC-POVM. In PBPT the set $\{P(B|\tilde{A}_k, I)\}$ has an analogous meaning, it describes the probabilities for the truth of proposition B given that each proposition A_k is considered to be true and non-contradictory. These probabilities summarise our knowledge about proposition B given our knowledge about propositions $\{A_k\}$. If we now chose to assign the values of $Q(\Pi_k|I)$ and $Q(\Sigma|\Pi_k)$ to the values of $P(A_k|I)$ and $P(B|\tilde{A}_k, I)$ respectively, then both rules Eqs. (27) and (33) lead to the same prediction for the

value of the probabilities $Q(\Sigma|\rho)$ and $P(B|\sum_{k=0}^{d^2-1} \tilde{A}_k, I)$. We have thus now two theories, Quantum mechanics and Paraconsistent bayesian probability theory, with differences in their origins and formalisms. Yet, with the proper identifications, these lead to the same predictions.

In recent years there has been an increasing interest in determining whether other theories might exhibit properties considered to be hallmarks of Quantum mechanics such as for instance interference, entanglement and incompatible measurements; this in the hope that Quantum mechanics can be deduced from these more general theories with the help of a suitable set of physically motivated constraints. For example, Spekken's toy model [13, 14] was constructed upon the knowledge balance principle. Although the model has many properties in common with Quantum mechanics, this does not emerge as a particular case of it. Generalised probabilistic theories [15] are based on a set of seven assumptions which allow to construct states and operations for composite systems which include those of Quantum mechanics. Here, we have introduced a new theory of probability based on the sole premise that its underlying logic must account for the possibility of contradictions. Thus, PBPT is a quantitative formulation of how to make rational decisions in presence of uncertainties and contradictions. This new theory is clearly of epistemological character, it says more about our believes on the plausibility of nature's behaviour than about nature itself, that is, PBPT does not describe the physical reality, it rather provides a set of rules, or algorithm, to calculate probabilities of certain propositions. Whether these propositions are of ontological or epistemological character cannot be decided within PBPT. In absence of contradictions PBPT reduces to BPT. Within PBPT and at hand of a simple model of propositions we deduced a total probability rule which agrees in form and meaning with the quantum mechanical one. This is remarkable, since we have only demanded a change of the underlying logic without resorting to any conception about physical reality. Nevertheless, this principle is not enough to single out a space state which agrees with the quantum mechanical one. In the case of Quantum mechanics operators representing states must be positive semidefinite, which imposes $d - 1$ constraints in the set of probabilities $Q(\Pi_k|I)$. These constraints do not arise within PBPT and thus additional principles are required.

Within the model of propositions leading to Eq. (33) we obtain that the probability of a contradiction is given by $P(\bar{A}^o|I) = 1/(d+1)$. This value equals the inner product among different members of the SIC-POVM, that is, $Tr(\Pi_k \Pi_l) = 1/(d+1)$. The relationship between nonorthogonality of quantum states and emergence of logical contradictions was noted by Birkhoff and von Neumann [16]. In order to solve the difficulties posed by these contradictions they proposed to change the rules of classical logic by modifying the distributive identities of

disjunction and conjunction. In our approach we do not attempt to eliminate logical contradictions but incorporate them into the formalism by means of paraconsistent logic, in particular the propositional calculus \mathcal{C}_1 . This choice is motivated by simplicity. Within this calculus all schemas and inference rules of classical propositional calculus are valid when replacing the negation by the strong negation and the consistency of the contradictoriness of any proposition always holds. These elements also appear in other propositional calculi, such as for example Paraconsistent Boolean algebra, and thus our results might hold for other paraconsistent logics.

Finally, we would like to mention that the connection between Quantum mechanics and Paraconsistent logic has been mentioned in the literature. In particular, this has been studied in the context of the superposition principle [17] and quantum computing [18]. Application of PBPT to the problem of inconsistent data basis is also feasible.

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- [1] E. T. Jaynes, *Probability Theory: The Logic of Science* (Cambridge, 2009).
- [2] B. de Finetti, *Theory of Probability* (New York: Wiley, 1970).
- [3] Mark A. Beaumont and Bruce Rannala, *Nat. Rev. Gen.* **5**, 251 (2004); J. S. Shoemaker, I. S. Painter and B. S. Weir, *Trends Genet.* **15**, 354 (1999).
- [4] Michael P. Hobson, Andrew H. Jaffe, Andrew R. Liddle, Pia Mukherjee, and David Parkinson (eds.), *Bayesian Methods in Cosmology* (Cambridge, 2014).
- [5] Newton C. A. da Costa, *Notre Dame Journal of Formal Logic* **15**, 497 (1974).
- [6] Newton C. A. da Costa in *Frontiers of Paraconsistent Logic* edited by D. Batens et al. (Hertfordshire, Research Studies Press, 2000).
- [7] G. Priest, R. Routley and J. Norman (eds.), *Paraconsistent Logic* (Munich, Philosophia Verlag, 1989).
- [8] Newton C. A. da Costa, Décio Krause and Otávio Bueno, in *Philosophy of Logic* edited by Dale Jacquette, *Handbook of the Philosophy of Science Vol. 5* (Elsevier BV, 2006).
- [9] M. Tribus, *Rational Descriptions, Decisions and Designs* (Pergamon Press, New York, 1969).
- [10] J. M. Renes, R. Blume-Kohout, A. J. Scott, and C. M. Caves, *J. Math. Phys.* **45**, 2171 (2004).
- [11] M. Born, *Zeits. Phys.* **37**, 863 (1926); **38**, 803 (1926).
- [12] C. A. Fuchs and R. Schack, *Rev. Mod. Phys.* **85**, 1693 (2013).
- [13] R. Spekkens, *Phys. Rev. A* **75**, 032110 (2007).

- [14] S. J. van Enk, Found. of Phys. **37** (2007).
- [15] J. Barrett, Phys. Rev. A **75**, 032304 (2007).
- [16] G. Birkhoff and J. von Neumann, Ann. Math. **37**, 823 (1936).
- [17] N. da Costa and C. de Ronde, Foundations of Physics **43**, 845 (2013).
- [18] J. C. Agudelo and W. Carnielli, J. Logic Computation **20** (2), 573 (2010).